

# Proton stability from a fourth family

CHRISTOPHER SMITH

*Université Lyon 1 & CNRS/IN2P3, UMR5822 IPNL,  
Rue Enrico Fermi 4, 69622 Villeurbanne Cedex, FRANCE*

## Abstract

The possibility to violate baryon or lepton number without introducing any new flavor structures, beyond those needed to account for the known fermion masses and mixings, is analyzed. With four generations, but only three colors, this minimality requirement is shown to lead to baryon number conservation, up to negligible dimension-18 operators. In a supersymmetric context, this same minimality principle allows only superpotential terms with an even number of flavored superfields, hence effectively enforces R-parity both within the MSSM and in a GUT context.

# 1 Introduction

The apparent stability of the proton is among the most puzzling phenomena, for which a compelling theoretical explanation still eludes us. On one hand, its lifetime is experimentally known to be greater than about  $10^{30}$  years [1]. Even compared to the age of the Universe, this is overly large. On the other hand, the conservation of the baryon ( $\mathcal{B}$ ) and lepton ( $\mathcal{L}$ ) numbers is purely accidental in the Standard Model (SM). It does not survive in many of its extensions, most notably in supersymmetry. So, given the extremely long proton lifetime, naturality seems to leave no other options than to forbid  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  couplings entirely, either by hand in a phenomenological context, or by a carefully designed theoretical setting in model-building approaches [2].

The decay of the proton is usually presented as the archetype of the processes induced by  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions. But this decay can also be regarded as a flavor transition, albeit of an extreme kind, since some quark flavors transmute into lepton flavors. Said differently, the  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions are flavored couplings. So, if they are not forbidden by some symmetries, it is their flavor structures which have to be very special given the experimental constraints from the proton lifetime and other  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  observables. This observation suggests that the origin of the proton stability could lie in the yet unknown mechanism from which all the known flavor structures, i.e. the quark and lepton masses and mixings, derive. The purpose of the present paper is to study whether this hypothesis is tenable. As such, it departs from most model-building approaches, for which the flavor structures of the  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions are not central.

In principle, some information on the dynamics going on above the electroweak scale is unavoidable to firmly establish a link between the flavor structures of the interactions conserving and violating  $\mathcal{B}$  and  $\mathcal{L}$ . But, even without a full-fledged dynamical flavor theory, it is possible to gather some information using low-energy effective theory techniques. Specifically, these tools allow us to study how large  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions could be if they are required not to introduce new flavor structures, beyond those already present in the SM. In some sense, this provides a natural scale for the  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions, against which the experimental constraints can be compared, as well as the predictions of a specific New Physics (NP) model.

This statement can be made more precise using the flavor-symmetry language. From a low-energy perspective, all the flavored couplings break the  $U(N_f)^5$  flavor symmetry of the SM gauge sector [3], with  $N_f$  the number of fermion flavors. The Yukawa couplings explicitly break  $SU(N_f)^5$  but not  $U(1)_{\mathcal{B},\mathcal{L}} \in U(N_f)^5$ , while  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions in general break  $U(N_f)^5$  completely. A natural scale for the  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions is obtained by preventing them from introducing any new breaking of the  $SU(N_f)^5$  part of  $U(N_f)^5$ . Indeed, this symmetry requirement does not necessarily forbid them, but forces them to be expressed entirely out of the Yukawa couplings, so that they inherit the very peculiar hierarchies observed in the quark and lepton masses and mixings. If these are sufficient to pass the experimental constraints, the proton decay puzzle would somehow be alleviated since the required fine-tunings of the  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  couplings could then be viewed as reminiscent of those present in the known flavor structures. Obviously, this would not yet be a solution to the proton decay puzzle, since a dynamical model enforcing such a restricted flavor-breaking sector is still missing, but it would nevertheless be a promising clue in that direction. In this respect, our approach is similar in spirit to Minimal Flavor Violation (MFV), see e.g. Refs. [4–6], but here applied to  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions [7–9].

In this work, the number of flavors is central for a simple combinatorial reason. As detailed in the following, the  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions necessarily involve epsilon contractions in flavor space since they are written as invariant under  $SU(N_f)^5$ , but not under  $U(N_f)^5$ . These epsilon tensors have

as many indices as there are generations. At the same time, Lorentz invariance requires an even number of fermion fields, and  $\Delta\mathcal{B} = \pm 1$  color singlets require three quark fields. So, going from three (odd) to four (even) generations completely changes the structure of the possible flavor-symmetric  $\Delta\mathcal{B}$  interactions. These interactions systematically require more quark fields, and thus get suppressed, with four generations.

The present work focuses on the flavor properties of the  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions, with or without an extra generation, but not on the phenomenology of such a new generation. In this respect, it should be noted that adding a generation of fermions is probably one of the simplest extensions of the SM (see e.g. Ref. [10] for reviews), and has recently received quite some attention for example in the context of the electroweak precision observables [11], in connection to the possible tensions exhibited in flavor physics [12, 13], for the discovery potential of colliders [14], or for baryogenesis [15]. Experimentally, very recent searches for the fourth-generation quarks have been performed at both Fermilab and at the LHC, producing mass bounds from the direct  $t'$  and  $b'$  production typically above 300-400 GeV [16] (though under some model-dependent assumptions about their couplings to light quarks, see e.g. Ref. [17]). This means that fourth-generation Yukawa couplings must be large,  $m_{t',b'}/v \gtrsim 2$ , with  $v \approx 246$  GeV the SM Higgs vacuum expectation value, and are thus pushed close to the threshold at which perturbative calculations would cease to make sense.

The SM Higgs searches also tightly constrain the presence of a heavy fourth generation, since it would enhance the Higgs production [18] (see also Ref. [19]). In this respect, the recent hint [20] of a SM-like Higgs boson at around 126 GeV is still inconclusive. Indeed, the  $\gamma\gamma$  signal is not much affected by a fourth generation, because the enhancement of the  $gg \rightarrow H$  production is mostly compensated by the smaller  $H \rightarrow \gamma\gamma$  rate [21], so better signals in alternative channels are compulsory. Finally, it should be stressed that the fourth generation discussed in the present work is purely sequential, i.e. its fermions have exactly the same quantum numbers as those of the first three generations, and get their masses from Yukawa couplings to the same SM Higgs field. Alternative extended fermionic contents (see e.g. Ref. [22, 23]) or more complicated Higgs sectors (see e.g. Refs. [19, 24]) could be devised to ease some of the experimental constraints, or preserve the perturbativity of the Lagrangian couplings. Though we will not discuss these models, our approach based on the flavor symmetry and its spurions is general enough, and could be easily adapted to study the flavor structures of  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions within these contexts.

The paper is organized according to the number and nature of the allowed  $SU(N_f)^5$  breaking terms, since their restriction constitutes the central working assumption used throughout this work. So, to systematically study their impact, and compare the resulting  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  operators in the three and four generation cases, we will introduce these breaking terms gradually. In the next section, only the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  gauge interactions are allowed, so that  $SU(N_f)^5$  is unbroken. Then, in the following sections, the complexity of the flavor-breaking sector is progressively increased by adding the SM Higgs sector and its Yukawa couplings, the neutrino masses using a seesaw model, the supersymmetric partners of SM particles, and finally, some Grand Unified Theory (GUT) boundary conditions.

## 2 $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge interactions

The SM gauge interactions are identical for all the generations of matter fields, denoted by the  $Q = (u_L, d_L)$ ,  $U = u_R^\dagger$ ,  $D = d_R^\dagger$ ,  $L = (\nu_L, e_L)$ , and  $E = e_R^\dagger$  left-handed Weyl spinors. At the Lagrangian level, nothing distinguishes the  $N_f$  flavors of the five fermion species, and a large  $U(N_f)^5$

global flavor symmetry is present. Since the  $U(1)$ s associated with  $\mathcal{B}$  and  $\mathcal{L}$  are linear combinations of the five  $U(1)$  factors of  $U(N_f)^5$ , and since we know that for such chiral symmetries, anomalies can arise, let us immediately restrict the flavor symmetry to  $G_F(N_f) = SU(N_f)^5$ , and see what are the possible  $G_F(N_f)$ -symmetric interactions violating  $\mathcal{B}$  and/or  $\mathcal{L}$ .

**With three generations,** only the  $G_F(3) \equiv SU(3)^5$  symmetric contractions  $\varepsilon^{IJK} X^I X^J X^K$  with  $X = Q, U, D, L, E$  have a nonzero  $\mathcal{B}$  or  $\mathcal{L}$  charge ( $I, J, K = 1, 2, 3$  are generation indices; repeated indices are summed over). An even number of such factors is needed to form a Lorentz scalar. No  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  invariant operators violating  $\mathcal{B}$  or  $\mathcal{L}$  can be constructed with six fields, so the leading  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  operators are dimension 18:

$$\mathcal{H}_{eff}^{gauge, SM3} = \frac{1}{\Lambda^{14}} ((LQ^3)^3 + (EU^2D)^3 + (EUQ^{\dagger 2})^3 + (LQD^{\dagger}U^{\dagger})^3 + h.c.), \quad (1)$$

where the flavor,  $SU(2)_L$ ,  $SU(3)_C$ , and Lorentz spinorial contractions are understood (only those contractions that maximally entwine the antisymmetric tensors do not vanish identically). These operators are all  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(3, 3)$ . Different patterns of  $\mathcal{B}$  and  $\mathcal{L}$  violation are possible but require at least six more fermion fields. For instance, with 18 fermion fields, dimension-27 operators inducing  $\pm(\Delta\mathcal{B}, \Delta\mathcal{L}) = (6, 0), (0, 6), (3, 9)$ , or  $(3, \pm 3)$  transitions can be written down.

At this level, in the absence of any flavor sector, it does not make much sense to discuss the phenomenology or a possible NP origin for these operators. Rather, the purpose of this construction is to point out a few basic features of the procedure used throughout the paper.

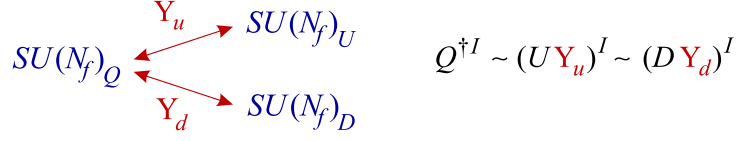
First, the  $G_F(3)$  symmetry requirement is very restrictive, since  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions are at least of dimension 18, and cannot induce proton decay. By contrast, imposing only the SM gauge invariance would allow for the dimension-six  $LQ^3$ ,  $EU^2D$ ,  $EUQ^{\dagger 2}$ , and  $LQD^{\dagger}U^{\dagger}$  effective interactions [25]. We thus immediately conclude that these interactions are not flavor-blind, i.e. require a nontrivial flavor structure to exist. Second, among the operators in Eq. (1), the one involving only  $SU(2)_L$  doublets,  $(LQ^3)^3$ , can be recognized as arising from the  $U(1)_{\mathcal{B}+\mathcal{L}}$  anomaly [26], i.e. from the nonperturbative SM gauge dynamics. Though enforcing only the  $G_F(3)$  symmetry requirement allows for additional operators in Eq. (1), it is nevertheless encouraging that it correctly predicts the order at which such effects are generated. Further, the SM itself thus provides an example of how the apparently peculiar  $\varepsilon^{IJK}$  contractions in flavor space can come into play. Third, inverting the argument, the anomalous nature of the  $U(3)^5$  symmetry justifies enforcing only  $SU(3)^5$ . Note, though, that from a flavor point of view, the  $(LQ^3)^3$  operators violate  $U(1)_Q \otimes U(1)_L$  but respect  $U(1)_U \otimes U(1)_D \otimes U(1)_E$ . So in principle, one can keep more than just  $SU(3)^5$  as exact.

**With four generations,** the  $\Delta\mathcal{L}$  or  $\Delta\mathcal{B}$  contractions invariant under  $G_F(4) \equiv SU(4)^5$  must involve four fields,  $\varepsilon^{IJKL} X^I X^J X^K X^L$ . The crucial difference with the three generation case is that for quarks, these monomials cannot be color singlets. Since the smallest common multiple of three and four is twelve, three factors of such quartic quark contractions are needed. The four-generation equivalents of the operators in Eq. (1) are thus of dimension 24:

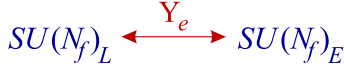
$$\mathcal{H}_{eff}^{gauge, SM4} = \frac{1}{\Lambda^{20}} ((LQ^3)^4 + (EU^2D)^4 + (EUQ^{\dagger 2})^4 + (LQD^{\dagger}U^{\dagger})^4 + h.c.), \quad (2)$$

and induce  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(4, 4)$ . Again, the one involving only  $SU(2)_L$  doublets originates from the  $\mathcal{B} + \mathcal{L}$  anomaly. However, another difference with the three generation case is that here, the  $G_F(4)$

Quark spurions:

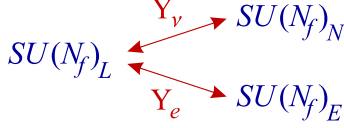


Massless neutrino spurions:



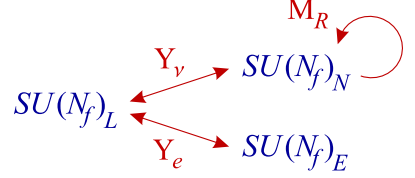
$$L^{\dagger I} \sim (E Y_e)^I$$

Dirac neutrino spurions:



$$L^{\dagger I} \sim (E Y_e)^I \sim (N Y_\nu)^I$$

Majorana neutrino spurions:



$$L^{\dagger I} \sim (E Y_e)^I \sim (L Y_\nu)^I$$

Figure 1: Spurions in the quark and lepton sectors, as needed to induce the known flavor structures. The Majorana mass term for the left-handed neutrino is assumed to originate from a seesaw mechanism of type I, i.e. from a heavy flavor-triplet of right-handed neutrinos. Those states are integrated out and do not appear at low energy, see Eq. (14).

symmetry is less restrictive, since lower-dimensional  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(0, 4)$  or  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(4, 0)$  operators can be constructed:

$$\mathcal{H}_{eff}^{gauge, SM4} = \frac{1}{\Lambda^{14}} ((LLE)^4 + (UDD)^4 + h.c.) . \quad (3)$$

This kind of couplings does not exist with three generations since they would involve an odd number of fermion fields. So, for the SM with four generations, the flavor symmetry fails at predicting the correct order at which  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  effects can arise from the nonperturbative SM gauge dynamics. Note, though, that if one imposes  $SU(4)^5 \otimes U(1)_U \otimes U(1)_D \otimes U(1)_E$ , the simplest interaction is again, trivially, the anomalous one.

As for three generations, the current flavor-blind setting is too restrictive to be relevant phenomenologically, and needs not be discussed further. Its purpose was first to introduce the crucial difference between enforcing  $G_F(3)$  or  $G_F(4)$  together with  $SU(3)_C$ , which stems from the mismatch between the number of colors and flavors, and second, to show that also with four generations, an operator can induce proton decay only if it has a nontrivial flavor structure, i.e. a nontrivial behavior not only under the  $U(1)_{\mathcal{B}, \mathcal{L}}$ , but also under  $G_F(4)$ .

### 3 Adding the Higgs sector

The Higgs sector breaks the  $G_F(N_f)$  flavor symmetry through the Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = U Y_u Q H_u + D Y_d Q H_d + E Y_e L H_d , \quad (4)$$

since those involve fermions transforming according to different  $SU(N_f)$ s. Note that in the present section, neutrinos are still massless, and the Yukawa couplings are written for a two-Higgs doublet

model of type II in anticipation of the Minimal Supersymmetric Standard Model (MSSM) discussion, but the SM case is trivially recovered through the identifications  $H_u \rightarrow H^*$ ,  $H_d \rightarrow H$ .

To systematically parametrize the impact of these breaking terms, the MFV strategy is first to promote them to spurions, so as to artificially restore the  $G_F(N_f)$  symmetry. Then, effective operators are constructed as formally invariant under  $G_F(N_f)$ , as well as under the gauge group  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ , using the fermion fields and the  $\mathbf{Y}_{u,d,e}$  spurions as building blocks. Once done, the  $\mathbf{Y}_{u,d,e}$  spurions are frozen back to their physical values,

$$v_u \mathbf{Y}_u = \mathbf{m}_u V, \quad v_d \mathbf{Y}_d = \mathbf{m}_d, \quad v_e \mathbf{Y}_e = \mathbf{m}_e, \quad (5)$$

with  $\mathbf{m}_{u,d,e}$  the  $N_f \times N_f$  diagonal mass matrices,  $V$  the  $N_f \times N_f$  generalized CKM matrix, and  $v_{u,d}$  the  $H_{u,d}^0$  vacuum expectation values. In this way, the specific flavor-breaking character of the Higgs sector, i.e. the transformation properties of its spurions as well as their hierarchical structures, is exported onto the effective operators.

Basically, the main feature of the Yukawa spurions is to interconnect the flavor  $SU(N_f)$ s, see Fig. 1. For example,  $Q^\dagger$ ,  $U\mathbf{Y}_u$ , and  $D\mathbf{Y}_d$  all transform in the same way under  $SU(N_f)_Q$ . So, enforcing  $G_F(N_f)$  no longer means constructing simple invariants like  $\varepsilon^{IJK} Q^I Q^J Q^K$  for  $N_f = 3$  or  $\varepsilon^{IJKL} Q^I Q^J Q^K Q^L$  for  $N_f = 4$ . This allows for simpler effective operators, since monomials transforming identically under  $G_F(N_f)$  need not have the same charges under the SM gauge group.

**With three generations,** and in the presence of the  $\mathbf{Y}_{u,d,e}$  spurions,  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  operators arise much earlier than at  $\mathcal{O}(\Lambda^{-14})$ . Since three lepton or three quark fields are needed to form a  $\Delta\mathcal{L} \neq 0$  or  $\Delta\mathcal{B} \neq 0$  but  $G_F(3)$ -invariant combination, the leading  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  invariant operators involve six fermion fields:

$$\mathcal{H}_{eff}^{Yukawa, SM3} = \frac{1}{\Lambda^5} (EL^{\dagger 2} U^3 + L^{\dagger 3} Q^\dagger U^2 + D^4 U^2 + D^3 U Q^{\dagger 2} + D^2 Q^{\dagger 4} + h.c.) , \quad (6)$$

and respect either  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(1, 3)$  or  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(2, 0)$ . All these operators require at least one  $\mathbf{Y}_{u,d,e}$  insertion to form  $G_F(3)$  singlets, for example (spinor,  $SU(2)_L$ , and color contractions are understood)

$$L^{\dagger 3} Q^\dagger U^2 = \varepsilon^{IJK} L^{\dagger I} L^{\dagger J} L^{\dagger K} \otimes \varepsilon^{LMN} Q^{\dagger L} (U\mathbf{Y}_u)^M (U\mathbf{Y}_u)^N + \dots , \quad (7a)$$

$$EL^{\dagger 2} U^3 = \varepsilon^{IJK} (E\mathbf{Y}_e)^I L^{\dagger J} L^{\dagger K} \otimes \varepsilon^{LMN} (U\mathbf{Y}_u)^L (U\mathbf{Y}_u)^M (U\mathbf{Y}_u)^N + \dots . \quad (7b)$$

Also, none of them breaks  $G_F(3)$  in the same way as Eq. (1), which remain the simplest operators satisfying  $\Delta\mathcal{B} = \Delta\mathcal{L}$ , even in the presence of the Yukawa spurions. In other words, and without surprise, the anomalous breaking of  $\mathcal{B} + \mathcal{L}$  does not spill over to lower dimensional operators. Still, it is interesting to note that the flavor contractions in Eq. (7) take place entirely in the  $SU(3)_Q \otimes SU(3)_L$  space, as for the anomalous operator of Eq. (1).

The  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(1, 3)$  operators can induce proton decay, but are very suppressed by the  $G_F(3)$  symmetry, the limited spurion content, and their high dimensionality. The former suppression comes from the need to extract only first-generation up quarks, and no bottom quark or tau lepton, while the epsilon antisymmetry asks for the flavors to be different. To compensate, one needs to insert the nondiagonal  $\mathbf{Y}_u$  as appropriate, and extract a  $\nu_\tau$  instead of a  $\tau$ . From Eq. (7), the piece of the  $L^{\dagger 3} Q^\dagger U^2$  operator contributing to proton decay ends up suppressed by  $(m_u/v_u)^2 V_{ub} \sim 10^{-13}$ , while that from the  $EL^{\dagger 2} U^3$  operator by  $(m_u/v_u)^3 V_{us} V_{ub} \sim 10^{-19}$ . This flavor suppression, combined with

the overall factor of the order of  $\mathcal{O}(m_{p^+}^{11}/\Lambda^{10})$  for the decay rate, ensures the proton lifetime is above about  $10^{30}$  years even for a relatively low NP scale,  $\Lambda \gtrsim 1$  TeV. This is sufficient since the bounds on the  $\Delta\mathcal{L} = \pm 3$  decay channels are much less tight than the best bound of  $8.2 \times 10^{33}$  years [1] for the  $\Delta\mathcal{L} = 1$  mode  $p^+ \rightarrow e^+ \pi^0$ , which cannot be induced by (6). These two suppression mechanisms similarly ensure that the  $\Delta\mathcal{B} = 2$  operators do not induce too rapid  $n - \bar{n}$  oscillations.

So, in the presence of the Higgs sector, the simplest  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions that are naturally non vanishing are dimension nine. They exhibit a strong hierarchy, inherited from the Yukawa couplings, which suffices to pass all the current experimental bounds.

**With four generations,** the simplest  $\Delta\mathcal{B}$  structures still require at least twelve quark fields (least common multiple of four flavors and three colors), as for the pure gauge situation. Proton decay as well as neutron oscillations are thus forbidden. Said differently, in the presence of a fourth generation, an operator inducing proton decay must have a flavor structure orthogonal to that of the Yukawa couplings. If a NP model does not allow for this, then the proton is stable.

By contrast, in the leptonic sector,  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  invariant operators with four lepton fields can be constructed,

$$\mathcal{H}_{eff}^{Yukawa, SM4} = \frac{1}{\Lambda^6} L^4 H_u^4 + \frac{1}{\Lambda^7} L^4 H_u^2 D U^\dagger + \mathcal{O}(\Lambda^{-8}) + h.c. , \quad (8)$$

which are  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(0, 4)$ . The first operator being totally symmetric under the exchange of the four  $SU(2)_L$  contractions  $(LH_u)$ , a large number of  $\mathbf{Y}_e$  insertions is needed:

$$L^4 H_u^4 = \varepsilon^{IJKL} (LH_u)^I (\mathbf{Y}_e^\dagger \mathbf{Y}_e LH_u)^J ((\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2 LH_u)^K ((\mathbf{Y}_e^\dagger \mathbf{Y}_e)^3 LH_u)^L + \dots , \quad (9)$$

resulting in a strong suppression by at least  $m_\mu^2 m_\tau^4 m_{\tau'}^6 / v_d^{12} \sim \mathcal{O}(10^{-8})$  for  $m_{\tau'} = 500$  GeV and  $\tan\beta = 3$  (which is close to the perturbativity limit with  $m_{\tau'}/v_d \approx 5$ ). For that reason, the second operator could be larger

$$L^4 H_u^2 D U^\dagger = \varepsilon^{IJKL} (LH_u)^I (\mathbf{Y}_e^\dagger \mathbf{Y}_e LH_u)^J (L^K L^L) (D \mathbf{Y}_d \mathbf{Y}_u^\dagger U^\dagger) + \dots \quad (10)$$

With  $J = 4$ , no light lepton mass factors are needed. Still, for  $\Lambda \gtrsim 1$  TeV, the high-dimensionality together with the fact that one of the external states is necessarily a heavy fourth generation lepton,  $\nu_{\tau'}$  or  $\tau'$ , makes this interaction difficult to probe experimentally.

At this stage, it must be emphasized that these numerical estimates are valid only in the perturbative regime. If the fourth generation is too heavy, the MFV expansions are not predictive. Indeed, let us remind that the standard MFV procedure relies not only on a restricted number of spurions, but also on a strong naturality principle. Typically, MFV leads to polynomial expansions in powers of the Yukawa couplings. For example, the expansion

$$a_0 \mathbf{1} + a_1 \mathbf{Y}_e^\dagger \mathbf{Y}_e + a_2 (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2 + \dots \sim \mathbf{1} \oplus (\mathbf{N}_f \otimes \bar{\mathbf{N}}_f) , \quad (11)$$

transforming as the  $SU(N_f)_L$  adjoint can be inserted in several places in Eqs. (9, 10). Importantly, all the numerical coefficients (the  $a_i$  of Eq. (11)) are required to be natural, i.e. of  $\mathcal{O}(1)$ . It is only in that case that the hierarchical structures of the Yukawa couplings are passed on to the other flavor couplings. But this is tenable only when the Yukawa couplings are also of  $\mathcal{O}(1)$  because for consistency,  $G_F$ -singlet traces like  $\langle \mathbf{Y}_e^\dagger \mathbf{Y}_e \rangle$  are implicitly absorbed into these coefficients (this also follows from the Cayley-Hamilton theorem, see Ref. [6]). So, if the Yukawa couplings ceased to

be perturbative, the naturality assumption collapses, the  $a_i$  become arbitrary, and MFV fails at transmitting the hierarchies of the Yukawa couplings to the other flavored couplings. This would be particularly problematic in the quark sector, since the MFV control over the  $\Delta\mathcal{B} = \Delta\mathcal{L} = 0$  FCNC operators of relevance in  $K$  and  $B$  physics would be lost.

For our purpose, even if MFV may lose its original motivation drawn from the tight experimental constraints on FCNC observables, this perturbativity issue is only marginal. Indeed, the impossibility to control the coefficients concerns only those operators allowed by the symmetry and spurion content. But, with four generations,  $\Delta\mathcal{B} = 1$  operators are forbidden from the start. As said, such an operator requires a new spurion to be allowed, and this spurion must transform intrinsically differently than the Yukawa spurions under  $G_F(4)$ .

A second issue is worth mentioning about the four-generation MFV implementation discussed here. Even if perturbativity survives, the  $G_F(N_f)$  version of MFV is much less predictive when  $N_f > 3$  since the background values (5) are not entirely fixed. In the four-generation case, the masses of the fourth-generation fermions as well as the extended CKM and PMNS parameters are unknown. So, the FCNC operators relevant for the tightly constrained  $K$  and  $B$  observables are not completely controlled. This was discussed in Ref. [13], where the  $G_F(4)$  version of MFV is projected onto a  $G_F(3)$  structure by factoring out a new  $G_F(3)$  spurion accounting for the fourth-generation degrees of freedom. The impact on the FCNC observables was then analyzed, as a function of the assumed flavor structure of this new spurion.

For our purpose, this predictivity issue is not directly relevant. First, the fourth generation fermions are not integrated out, but occur as dynamical fields in the effective operators. The full  $G_F(4)$  flavor symmetry of the gauge sector is active, and there is no need to project onto  $G_F(3)$ . Second, even if the unknown  $G_F(4)$  spurion parameters render numerical estimates uncertain, like for the operators in Eqs. (9, 10), they do not play any role in the construction of the  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  effective operators, which is entirely fixed by the  $G_F(4)$  symmetry properties of the spurions.

## 4 Adding neutrino masses

In the previous section, neutrinos were left massless, as in the minimal version of the Standard Model. The simplest way to give them a mass is to add the Yukawa coupling

$$\mathcal{L}^{\text{Dirac}} = N\mathbf{Y}_\nu LH_u, \quad (12)$$

to Eq. (4), with  $N = \nu_R^\dagger$  a flavor  $N_f$ -plet of right-handed neutrinos (see Fig. 1). However, neither the presence of  $\mathbf{Y}_\nu$  among the spurions nor the possibility to write operators involving  $N$  fields change the previous conclusions. Obviously,  $\Delta\mathcal{B}$  operators are unaffected by the spurion content of the leptonic sector. Further,  $\mathcal{L}$  can still change only in step of  $N_f$ , so the simplest operators have the same  $\mathcal{B}$  and  $\mathcal{L}$  overall charges as in Eq. (6) or (8), but for the new  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(0, 6)$  operator  $N^6$  permitted in the three generation case.

The situation changes if left-handed neutrinos have a Majorana mass term, as arising from the  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(0, 2)$  dimension-five Weinberg operator:

$$\mathcal{H}_{eff}^{\text{Majorana}} = (LH_u)^T \frac{\mathbf{m}_\nu}{v_u^2} (LH_u). \quad (13)$$

Such an effective interaction typically arises from the type-I seesaw mechanism [27], which relates  $\mathbf{m}_\nu$



to the right-handed neutrino Majorana mass  $\mathbf{M}_R$  and neutrino Yukawa couplings  $\mathbf{Y}_\nu$  as

$$\mathbf{Y}_\nu^\dagger \equiv \frac{\mathbf{m}_\nu}{v_\nu} = v_u \mathbf{Y}_\nu^T \mathbf{M}_R^{-1} \mathbf{Y}_\nu . \quad (14)$$

(the  $\dagger$  is introduced only for future convenience.) In this way, one avoids introducing unnaturally small Lagrangian couplings in the neutrino sector since  $\mathbf{Y}_\nu$  can be of  $\mathcal{O}(1)$  when  $\mathbf{M}_R \approx 10^{13}$  GeV.

For simplicity, in the present analysis, we add only the effective  $\mathbf{Y}_\nu$  spurion to  $\mathbf{Y}_{u,d,e}$ . This spurion is symmetric, transforms as  $\mathbf{N}_f \otimes \mathbf{N}_f$  under  $SU(N_f)_L$ , but has tiny background values, with entries at most of  $\mathcal{O}(10^{-12})$  for the first three generations. On the other hand, the spurion combination  $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ , characteristic of an underlying seesaw mechanism, is not suppressed<sup>1</sup> but irrelevant for the construction of  $\Delta\mathcal{L}$  operators since it transforms as  $\mathbf{Y}_e^\dagger \mathbf{Y}_e$ , and will thus be left out.

**With three generations,** there is no more need to go fetch the  $\Delta\mathcal{L} = 3$  terms, as in Eq. (6), because  $\Delta\mathcal{L} = 1$  interactions are allowed through the generic contraction  $\varepsilon^{IJK}(\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{IJLK}$  (the  $\mathbf{Y}_e^\dagger \mathbf{Y}_e$  insertion is needed to compensate for the symmetry<sup>2</sup> of  $\mathbf{Y}_\nu$ ). So, the simplest interactions are the dimension-six  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(1, 1)$  Weinberg operators [25]

$$\mathcal{H}_{eff}^{Seesaw, SM3} = \frac{1}{\Lambda^2} (LQ^3 + EU^2D + EUQ^{\dagger 2} + LQD^\dagger U^\dagger + h.c.) . \quad (15)$$

These operators cannot be trivially discarded since  $\mathcal{B} + \mathcal{L}$  is not a good symmetry. In other words, they have the same  $U(1)$  charges (modulo three) as the operators (1). Including the Higgs fields,  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(1, -1)$  dimension-seven operators can be constructed (still proportional to  $\mathbf{Y}_\nu$ ), along with corrections to the Majorana mass term for the neutrinos.

It is only once neutrinos get a Majorana mass term that MFV permits to set a nontrivial natural scale for the Wilson coefficients of the Weinberg operators. But, neutrinos being so light, these couplings will automatically be very suppressed. A further suppression comes from the epsilon contractions, exactly like for the operators in Eq. (7), since their antisymmetry asks for the flavors of the quark fields to be all different, while only  $u, d, s$  can be present to induce proton decay. In the Appendix, we explicitly show that numerically, these two mechanisms are sufficient to pass the proton decay bounds even for a relatively low NP scale,  $\Lambda \gtrsim 10$  TeV. As discussed there, it may even be possible to pass these bounds at the electroweak scale, provided Yukawa coupling insertions are accompanied by gauge couplings and loop factors.

**With four generations,** the situation is again very different. The presence of  $\mathbf{Y}_\nu$  in the leptonic sector does not change anything for  $\Delta\mathcal{B}$  operators, which still need twelve quark fields, as in Eqs. (2, 3). Thus, proton decay and neutron oscillations remain forbidden.

In the leptonic sector,  $\mathbf{Y}_\nu$  brings in an even number of  $SU(4)_L$  flavor indices, so its main impact is to allow for new  $\Delta\mathcal{L} = \pm 2$  operators (operators with  $H_u \leftrightarrow H_d^\dagger$  are understood):

$$\mathcal{H}_{eff}^{Seesaw, SM4} = \frac{\varepsilon^{IJKL}}{\Lambda} (\mathbf{Y}_\nu)^{IJ} (LH_u)^K (LH_u)^L + \frac{\varepsilon^{IJKL}}{\Lambda^3} (\mathbf{Y}_\nu)^{IJ} (LH_u)^K (LH_u)^L H_u H_d \quad (16)$$

$$+ \frac{\varepsilon^{IJKL}}{\Lambda^3} (\mathbf{Y}_\nu)^{IJ} L^K L^L H_u (D\mathbf{Y}_d Q + E\mathbf{Y}_e L + Q^\dagger \mathbf{Y}_d^\dagger U^\dagger) + \mathcal{O}(\Lambda^{-5}) + h.c. . \quad (17)$$

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<sup>1</sup>This spurion combination is mostly relevant for lepton flavor violating effects, since  $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$  and  $\mathbf{Y}_e^\dagger \mathbf{Y}_e$  are not simultaneously diagonal. The current experimental bounds on the lepton flavor violating transitions like  $\ell^I \rightarrow \ell^J \gamma$ ,  $I \neq J$ , severely limit these off-diagonal entries for NP scales at or below the TeV scale, see e.g. Ref. [5].

<sup>2</sup>There is no singlet in  $\mathbf{3} \otimes \mathbf{6} = \mathbf{10} \oplus \mathbf{8}$ , but  $\mathbf{Y}_e^\dagger \mathbf{Y}_e \sim \mathbf{8}$  permits to extract the one occurring in  $\mathbf{3} \otimes (\mathbf{6} \otimes \mathbf{8})$ .

The dimension-five operator contributes to the Majorana mass term itself. But since it requires some Yukawa insertions to be nonzero ( $\Upsilon_\nu$  and  $(LH_u)^2$  are symmetric in flavor space), the induced effect on  $\mathbf{m}_\nu$  is subleading. The operators in the second line contribute to  $\Delta\mathcal{L} = \pm 2$  flavor transitions like for example  $K^+ \rightarrow \pi^- \ell^+ \ell^+$ , but are presumably too suppressed by the several factors of the light fermion mass to be accessible experimentally, even for large fourth-generation entries in  $\Upsilon_\nu$  [28]. In this respect, let us stress that the same provision on the perturbativity of the spurions as in the previous section is required for MFV expansions to be valid. As before, this is irrelevant for proton decay, since no expansion can be written for  $\Delta\mathcal{B} = \pm 1$  couplings, given the assumed spurion content.

## 5 Adding supersymmetric partners

Up to now, an even number of matter fields was required to form Lorentz invariant. This no longer holds when those fields receive scalar superpartners, with which they share their quantum numbers. The consequence is well known: the MSSM a priori allows for the renormalizable  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions [29]

$$\mathcal{W}_{\Delta\mathcal{B},\Delta\mathcal{L}} = \frac{1}{2}\lambda^{IJK}L^IL^JE^K + \lambda'^{IJK}L^IQ^JD^K + \mu'^IH_uL^I + \frac{1}{2}\lambda''^{IJK}U^ID^JD^K, \quad (18)$$

where  $Q, U, D, L, E$  now denote superfields (the corresponding soft-breaking terms are understood throughout this section). With these couplings of order one, the proton would decay very quickly. So, phenomenologically, they are usually discarded by enforcing by hand a new discrete symmetry, R-parity [30].

The R-Parity Violating (RPV) couplings are flavored: they not only break  $U(1)_{\mathcal{B}}$  and  $U(1)_{\mathcal{L}}$ , but also  $SU(N_f)^5$ . So, following the philosophy exposed in the Introduction, a natural size for these couplings can be derived by forbidding them from introducing any new flavor structure. In other words, their natural size is obtained by expressing them as  $G_F(N_f)$  invariant combinations of the  $\mathbf{Y}_{u,d,e}$  and  $\Upsilon_\nu$  spurions. The main phenomenological consequence can immediately be guessed from the previous sections. Indeed, integrating out the sparticles, the RPV couplings generate precisely the effective  $\Delta\mathcal{B}$  and/or  $\Delta\mathcal{L}$  operators studied before. Since in addition the same minimal spurion content is assumed, we immediately know that the proton decays sufficiently slowly in the three-generation case, and is absolutely stable in the four-generation case, where only  $\Delta\mathcal{B} = \pm 4$  operators can be constructed.

At this stage, one may be a bit puzzled by the asymmetric status given to the R-parity conserving and violating couplings. After all, they are all renormalizable couplings of the MSSM superpotential<sup>3</sup>. The procedure we follow can be interpreted in several ways:

- From a purely low-energy perspective, there is no reason to expect the RPV couplings to be flavor blind (e.g.,  $\lambda^{IJK} = \lambda$  for all  $I, J, K$ ), especially given the highly hierarchal SM flavor structures. So, writing them down in terms of the known flavor structures,  $\mathbf{Y}_{u,d,e}$  and  $\Upsilon_\nu$ , is an attempt at assigning to them a similarly nonuniversal flavor structure in a controlled way.
- Taking the opposite view, it is obvious that assigning by hand a sufficiently peculiar  $SU(N_f)^5$  flavor structure to each RPV coupling can suppress the proton decay to an acceptable rate.

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<sup>3</sup>Further, they get mixed by the possible redefinitions needed to get physical fields out of the Lagrangian  $H_d$  and  $L$  fields. The consistency of the MFV expansion in the presence of this freedom was analyzed in details in Ref. [8].

However, it is generally believed that such assignments are too fine-tuned to be acceptable, and consequently, that only an exact symmetry can prevent rapid proton decay. The present analysis shows that the fine-tunings of the RPV couplings are in reality no less natural (or less unnatural) than the strong hierarchies exhibited by the known quark and lepton masses and mixings.

- From a high-energy perspective, it is reasonable to expect that none of the low-energy flavor couplings is fundamental. But, if they all derive from a limited set of fundamental high-energy spurions, they should be related since they are ultimately redundant. Though a full-fledged dynamical theory of flavor would be required to consistently implement this picture, the MFV procedure can be seen as an attempt at capturing such relationships. So, under the assumption that the low-energy flavor couplings are (maximally) redundant, expressing the unknown RPV couplings in terms of the known quark and lepton masses and mixings does not presume anything about their relative fundamentality.
- Finally, one can also consider the present section as a kind of consistency check for the previous, purely effective discussions. It illustrates how enforcing MFV on a well-known NP model with  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions leads to the suppression obtained in a model-independent way.

To a large extent, one can recognize in the above points an exact translation in the MSSM context of the general strategy presented in the Introduction. These points were also discussed in Ref. [7], dedicated exclusively to the consequences of MFV for the RPV couplings.

Let us now particularize the discussion to the three and four generation case, and examine the phenomenological consequences.

**With three generations,** the minimal spurion content allows for all the RPV couplings. This construction was performed in detail in Ref. [7], to which we refer for more information.

Briefly, the  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(1, 0)$  RPV coupling is expressible entirely out of  $\mathbf{Y}_{u,d}$ , for example as

$$\begin{aligned} \lambda_{MSSM3}^{IJK} = & \lambda_1'' \varepsilon^{LJK} (\mathbf{Y}_u \mathbf{Y}_d^\dagger)^{IL} + \lambda_2'' \varepsilon^{IMN} (\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{JM} (\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{KN} + \lambda_3'' \varepsilon^{LMN} \mathbf{Y}_u^{IL} \mathbf{Y}_d^{JM} \mathbf{Y}_d^{KN} \\ & + \lambda_4'' \varepsilon^{LMN} \varepsilon^{IAB} \varepsilon^{DJK} \mathbf{Y}_d^{\dagger LD} \mathbf{Y}_u^{MA} \mathbf{Y}_u^{\dagger NB} + \dots, \end{aligned} \quad (19)$$

with  $\lambda_i'' \sim \mathcal{O}(1)$ . So,  $\lambda''$  can be large, up to  $\mathcal{O}(1)$  values for  $\lambda''^{312}$  when  $\tan\beta = v_u/v_d \gtrsim 20$ , but also show a strong hierarchy. On the other hand, the leptonic Yukawa couplings  $\mathbf{Y}_e$  and  $\mathbf{Y}_\nu$  permit to change  $\mathcal{L}$  only in step of three. So, the  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(0, 1)$  RPV couplings require the presence of a Majorana mass term for the neutrinos, for example as

$$\mu_{MSSM3}^I = \mu_1' \varepsilon^{IJK} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{JK} + \dots, \quad (20)$$

$$\lambda_{MSSM3}^{IJK} = \lambda_1 \varepsilon^{ILM} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{LM} \mathbf{Y}_e^{KJ} + \lambda_2 \varepsilon^{IMJ} (\mathbf{Y}_e \mathbf{Y}_\nu)^{KM} + \dots, \quad (21)$$

$$\lambda_{MSSM3}^{IJK} = \lambda_1' \varepsilon^{ILM} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{LM} \mathbf{Y}_d^{KJ} + \dots, \quad (22)$$

with  $\mu_i$ ,  $\lambda_i$ , and  $\lambda_i'$  of  $\mathcal{O}(1)$ . Note that  $\mathbf{Y}_e^\dagger \mathbf{Y}_e$  factors have been introduced to compensate for the symmetry of  $\mathbf{Y}_\nu$ . So, these RPV couplings are all suppressed by the tiny neutrino masses, as well as by some charged lepton masses.

The suppression of the RPV couplings brought in by the proportionality to  $\mathbf{Y}_\nu$  and by the anti-symmetry of the epsilon contractions is sufficient to pass the bounds on proton decay (as well as on

all other  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  observables, see Ref. [7]). This can be understood from the numerical analysis of the Weinberg operators presented in the Appendix. Indeed, integrating out the squarks and slepton, the only effective operator accessible from the couplings of Eq. (18) is  $L^I Q^J D^{\dagger L} U^{\dagger K}$ , proportional to  $\lambda'^{IJM} \lambda''^{KLM}$ . Under MFV, this operator ends up much more significantly suppressed than  $LQ^3$  because light-quark mass factors necessarily occur (see Eq. (29)). So, the scale  $\Lambda$ , now identified with the  $D^M$  mass, can be of a few hundred GeV. The only new feature compared to the Appendix is that the predicted rates show a strong dependence on  $\tan\beta$ . If it is large, the current proton decay bounds require relatively heavy squarks, of at least a few TeV [7]. Note, finally, that if a holomorphic constraint is imposed on the spurions, as proposed recently in Ref. [9], even this operator becomes forbidden, and the leading proton decay mechanism is induced only at the loop level by the  $\lambda^{IJK}$  and  $\lambda''^{IJK}$  couplings. Current experimental bounds are then easily satisfied, even for light sparticles.

In addition to the superpotential terms (18), nonrenormalizable interactions may be present since the MSSM is most probably only an effective low-energy theory. Those have the same forms as the effective operators discussed in the previous sections, up to the replacement of SM fermionic fields by their corresponding superfields, and the flavor symmetry requirements are identical. Consider for example the dimension-four superpotential terms of the form (15) which can induce proton decay [31]. Those actually conserve R-parity, so it is only at the loop level, with the help of gauge interactions, that they contribute to proton decay. With these gauge couplings and loop factors, the proportionality to  $\Upsilon_\nu$ , and the antisymmetric contractions, proton decay bounds are easily satisfied, even for relatively low NP scales [7].

**With four generations,** none of the  $\mathcal{W}_{\Delta\mathcal{B},\Delta\mathcal{L}}$  terms are allowed,

$$\mu'^I_{MSSM4} = \lambda^{IJK}_{MSSM4} = \lambda'^{IJK}_{MSSM4} = \lambda''^{IJK}_{MSSM4} = 0, \quad (23)$$

simply because there is no way to contract an odd number of flavor indices using the available spurions (two indices) and the epsilon tensors (four indices). In other words, the transformation properties of the RPV couplings under  $SU(4)^5$  are orthogonal to those of the known flavor structures. So, the minimal spurion content effectively enforces R-parity, the proton decay is forbidden, and the lightest supersymmetric particle is stable.

The consequences for higher dimensional operators, if present, can be readily drawn from the previous sections. The  $\Delta\mathcal{B}$  couplings require no less than twelve quark superfields, so  $\mathcal{B}$  is effectively conserved. Also at the effective level, neither proton decay nor neutron oscillations are permitted. On the other hand,  $\mathcal{L}$  can be violated, but exclusively by an even number. So, R-parity emerges naturally also for nonrenormalizable operators, making it a good effective symmetry.

In addition, the  $\Delta\mathcal{L} = 2$  operators are suppressed by neutrino masses, so the dominant  $\Delta\mathcal{L}$  effects arise from the superfield version of the  $\Delta\mathcal{L} = \pm 4$  operators of Eq. (8). As discussed there, those are significantly suppressed by the NP scale, require at least four leptons, and thus should have no impact on low-energy phenomenology.

## 6 Adding GUT boundary conditions

A crucial feature of the spurions is their separation into two sectors, leptonic and baryonic, see Fig. 1. The absence of connection between the  $SU(N_f)_Q \otimes SU(N_f)_U \otimes SU(N_f)_D$  and the  $SU(N_f)_L \otimes SU(N_f)_E$  flavor spaces effectively factorizes the conservation or nonconservation of  $\mathcal{B}$  and  $\mathcal{L}$ . For instance, with three generations,  $\Delta\mathcal{B} = \mathbb{Z}$  and  $\Delta\mathcal{L} = 3\mathbb{Z}$  interactions are allowed by the Yukawa couplings, but

$\Delta\mathcal{L} = \mathbb{Z}$  interactions are proportional to the neutrino Majorana masses. With four generations, only  $\Delta\mathcal{B} = 4\mathbb{Z}$  and  $\Delta\mathcal{L} = 2\mathbb{Z}$  are allowed ( $\Delta\mathcal{L} = 4\mathbb{Z}$  without neutrino Majorana masses). These “selection rules”, when combined with the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  invariance, strongly constrain the Wilson coefficients of  $\Delta\mathcal{B}$ ,  $\Delta\mathcal{L}$  effective operators, or the RPV couplings of the MSSM.

In the present section, the goal is to illustrate what could occur if such a factorization does not survive beyond the SM or MSSM. To connect these two sectors realistically, GUTs like  $SU(5)$  or  $SO(10)$  appear tantalizing, especially given our focus on proton decay. At the same time, the GUT context with its rather precise dynamics and flavor structure represents a challenge for MFV, both at the conceptual and practical level. Let us comment on four particular issues:

1. **Generic MFV implementation in GUT:** The flavor group above the unification scale is smaller than  $SU(N_f)^5$ , since there are less fermion species [32]. For example, in  $SU(5)$ , there are two fermion multiplets per generation,  $\bar{\mathbf{5}}$  and  $\mathbf{10}$ , so the flavor group is  $SU(N_f)_{\bar{\mathbf{5}}} \otimes SU(N_f)_{\mathbf{10}}$ . Similarly, in  $SO(10)$ , each generation of fermions is put in a  $\mathbf{16}$  representation, and the flavor group is simply  $SU(N_f)_{\mathbf{16}}$ . Further, the spurion structure does not match the SM one. For example, in  $SU(5)$ , the spurions  $\mathbf{Y}_{10} \sim (\mathbf{1}, \mathbf{N}_f \otimes \mathbf{N}_f)$  and  $\mathbf{Y}_{\bar{\mathbf{5}}} \sim (\bar{\mathbf{N}}_f, \mathbf{N}_f)$  are not bifundamental representations of the SM flavor group  $SU(N_f)^5$ . As a result, matching an  $SU(N_f)_{\bar{\mathbf{5}}} \otimes SU(N_f)_{\mathbf{10}}$  MFV expansion onto an  $SU(N_f)^5$  expansion, the quark Yukawa couplings end up in the leptonic sector, and vice versa. Further, naturality may be lost in this projection because the coefficients of the  $SU(N_f)^5$  expansion are not of  $\mathcal{O}(1)$  in general. Even though the RGE tend to bring generic expansions back to MFV [6], whether this is always sufficient to recover naturality at low energy is not yet established (especially for  $N_f \neq 3$ , see point 3 below). Finally, to make matters worse, the fermion mass unification is delicate, and requires extending the minimal GUT spurion content. Besides the increased complexity of the MFV expansions, their predictivity is seriously affected because the background values of these spurions are not entirely fixed [32].
2. **Proton decay through GUT gauge interactions:** In GUT, baryon and lepton numbers are no longer good quantum numbers since quarks and leptons are unified. Said differently,  $U(1)_B$  and  $U(1)_L$  are not contained in  $U(N_f)_{\bar{\mathbf{5}}} \otimes U(N_f)_{\mathbf{10}}$  or  $U(N_f)_{\mathbf{16}}$ . Further, the gauge interactions induce proton decay through leptoquark tree-level exchanges. In the MFV language, the leptoquark gauge couplings become, after the spontaneous breaking down to the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  gauge group, genuine new  $SU(N_f)^5$  spurions connecting the leptonic and hadronic flavor spaces. Those allow the dimension-six  $(\Delta\mathcal{B}, \Delta\mathcal{L}) = \pm(1, 1)$  effective operators of Eq. (15), with the scale  $\Lambda = M_{GUT}$  set by the leptoquark gauge boson masses. MFV has no handle on these spurions, since they are protected by the GUT gauge symmetry, and thus cannot be assumed related to the Yukawa spurions in any way. So, one has to sufficiently increase the leptoquark masses, i.e. the unification scale. This does not invalidate the MFV framework per se, since the high unification scale required by proton decay is in any case (roughly) compatible with that obtained from the gauge coupling unification, but it shows its limitation. When proton decay is induced by the GUT gauge interactions, MFV is entirely irrelevant.
3. **Perturbativity up to the GUT scale:** As said, proton decay requires  $M_{GUT}$  to be rather close to the Planck scale. At the same time, if  $N_f > 3$ , the Yukawa couplings of the last  $N_f - 3$  generations have to be close to the perturbativity limit already at the electroweak scale, and certainly do not remain perturbative up to the GUT scale [23, 33, 34]. This is particularly serious in a supersymmetric setting, since the Higgs mass requires  $\tan\beta$  not too small but the current bound on  $m_{h'}$  is already above the electroweak scale [1]. Though it may actually

become a welcome feature in the context of the electroweak symmetry breaking [35], the onset of a nonperturbative regime not far from the TeV scale may render the GUT setting difficult to manage in practice. If one insists on perturbativity, one way to proceed is to alter the particle content, so as to modify the evolution up to the GUT scale [23]. But then, to have an impact, the extra particles should couple to quarks and leptons, i.e. should introduce new flavor couplings. Whether these couplings have to be part of the spurions or can be constructed out of the Yukawa spurions, the consequences for the MFV expansions, for proton decay, and for FCNC constraints, have to be analyzed on a case-by-case basis.

4. **Supersymmetric GUT and R-parity:** Even if the proton decay mechanism through gauge interactions is controlled by pushing the unification scale close to the Planck scale, this may not suffice. Indeed, in a supersymmetric setting, a resurgence of the RPV couplings (18) is possible, either directly from specific GUT-scale couplings, or concurrently with the GUT spontaneous breaking chain. For example, in  $SU(5)$ , the following trilinear coupling can be constructed

$$\mathcal{W}_{RPV} = \lambda_5^{IJK} \bar{5}^I \bar{5}^J 10^K, \quad (24)$$

which collapses to  $\lambda$ ,  $\lambda'$ , and  $\lambda''$  at low-energy. With thus both  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  couplings of similar sizes, and no suppression from the GUT scale since these superpotential couplings are renormalizable, the only way to prevent a too fast proton decay is to require  $\lambda_5^{IJK}$  to be tiny [36]. In  $SO(10)$ , the situation is a bit different because the  $\mathcal{B} - \mathcal{L}$  violating interactions are forbidden by the gauge symmetry. Though not automatic, R-parity can arise unscathed from the spontaneous symmetry breaking chain down to the SM gauge group [37].

Resolving all these issues would require a thorough and dedicated study of MFV in a GUT setting, and is clearly beyond our scope. As said, the goal here is to illustrate what happens when the leptonic and hadronic flavor spaces are connected, and point out where MFV could retain some usefulness in connection with the stability of the proton. So, we concentrate exclusively on the fourth issue above.

**With three generations,** RPV terms may be immediate to construct in terms of the Yukawa couplings. For instance, in  $SU(5)$ , we can write the coupling of Eq. (24) as

$$\lambda_5^{IJK} = \varepsilon^{IJL} (\mathbf{Y}_5)^{LK}. \quad (25)$$

Since  $\mathbf{Y}_5$  is related to  $\mathbf{Y}_{d,e}$ ,  $\lambda_5^{IJK}$  is not particularly suppressed, and the induced RPV couplings at the MSSM scale are way too large. In this case, the MFV construction fails to prevent a too fast proton decay. This can be traced back to the reduced flavor group, which imposes the coherences  $SU(3)_Q \otimes SU(3)_U \otimes SU(3)_E \rightarrow SU(3)_{10}$  and  $SU(3)_L \otimes SU(3)_D \rightarrow SU(3)_5$ . Indeed, when  $L$  and  $D$  (or  $Q$ ,  $U$ , and  $E$ ) are no longer allowed to transform independently under the flavor group, a coupling like  $\lambda^{IJK} L^I Q^J D^K$  can be constructed using only the Yukawa couplings, e.g. as  $\varepsilon^{IJK} L^I (\mathbf{Y}_d Q)^J D^K$  (compare with Eq. (25)). The need for a neutrino Majorana mass is circumvented, so the  $\Delta\mathcal{L} = \pm 1$  and  $\Delta\mathcal{B} = \pm 1$  couplings are equally large. The lifetime of the proton cannot be naturally explained by the flavor structure of the  $\lambda_5^{IJK}$  coupling, and some  $U(1)$ s have to be imposed.

On the contrary, in  $SO(10)$ , a restriction of the allowed flavor structures can effectively enforce R-parity. Indeed, if fermion masses derive only from Yukawa couplings to Higgses in the **10** and **126** representations, the only available spurions transform as **6**s under  $SU(3)_{16}$ , and do not permit to construct superpotential RPV terms. Of course, this observation is relevant only when the assumed

breaking scheme does not automatically lead to R-parity at low energy [37], so for example when  $\mathcal{B} - \mathcal{L}$  gets broken by a Higgs in the **16** representation.

From these two examples, it is clear that MFV is far less powerful in a GUT setting, but could still prove useful in some scenarios. So, the main message is that one should keep an eye on the transformation properties of the various flavor-symmetry breaking terms, as they could hold the key to proton stability.

**With four generations,** there is no way to form flavor-symmetric combinations of an odd number of matter superfields since the spurions and the  $SU(4)$  invariant tensors all have an even number of indices. For example, in  $SU(5)$ , the minimal spurion content forbids the coupling of Eq. (24),

$$\lambda_5^{IJK} = 0. \quad (26)$$

The same holds in  $SO(10)$ , for both renormalizable and nonrenormalizable couplings inducing a low-energy R-parity violation. The only way to violate Eq. (26) is to allow for a spurion with an odd number of flavor indices. But with the minimal matter content, this spurion would necessarily violate R-parity, so by assumptions, it is forbidden. Said differently, the couplings inducing RPV effects and those inducing the SM Yukawa couplings are entirely decoupled when there are four generations. They transform in radically different ways under the unified flavor group. So, the latter cannot be used to set a natural scale for the former.

As discussed before, nonperturbative Yukawa couplings would not invalidate Eq. (26) since it is simply forbidden. But, let us stress, if for some other reasons, perturbativity is nevertheless enforced by extending the matter content, there is no guarantee that Eq. (26) holds. Specifically, if a coupling with an odd number of flavor indices is needed, it can never be expressed in terms of the Yukawa couplings, hence must be part of the spurion content. But then, it would immediately set a nontrivial natural scale for  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  couplings. Given the tight experimental bounds, either this new spurion is particularly fine-tuned, or R-parity has to be enforced by some other means.

## 7 Conclusion

In the present paper, the flavor structures of the baryon and lepton number violating couplings have been systematically analyzed. The main technique was to relate their flavor structures to those needed to account for the quark and lepton masses and mixings, using the flavor-symmetry breaking language of minimal flavor violation. Our results can be split into those relevant for three and four generations:

**With three generations,** the minimal flavor structures needed to account for the SM fermion masses and mixings are sufficient to set a natural scale for all the effective  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions invariant under  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  (see Table 1). The Yukawa couplings  $\mathbf{Y}_u$ ,  $\mathbf{Y}_d$ , and  $\mathbf{Y}_e$  permit to construct  $\Delta\mathcal{B} = \pm 3/N_c$  and  $\Delta\mathcal{L} = \pm 3$  effective operators (with  $N_c = 3$  the number of QCD colors), while  $\Delta\mathcal{L} = \pm 1$  operators must be proportional to the neutrino Majorana mass. In some sense, this implements a picture where  $\Delta\mathcal{L} = \pm 1$  effects occur concurrently with the seesaw mechanism. Numerically, because of the smallness of the neutrino masses, and of the large hierarchies present in the Yukawa couplings, the Wilson coefficients of all the operators inducing proton decay are sufficiently suppressed to pass the tight experimental bounds (see the Appendix for an analysis of the dimension-six Weinberg operators (15)), even for TeV-scale new physics, provided of course this new physics does not introduce any new flavor structure.

	Spurions	Three flavors		Four flavors	
		Dim.	$\pm(\Delta\mathcal{B}, \Delta\mathcal{L})$	Dim.	$\pm(\Delta\mathcal{B}, \Delta\mathcal{L})$
SM gauge	–	27	$(\mathbf{6}, \mathbf{0}), (\mathbf{0}, \mathbf{6}), (\mathbf{3}, \mathbf{9}), (\mathbf{3}, \pm\mathbf{3})$	24	$(\mathbf{4}, \mathbf{4})$
		18	$(\mathbf{3}, \mathbf{3})$	18	$(\mathbf{4}, \mathbf{0}), (\mathbf{0}, \mathbf{4})$
Higgses	$\mathbf{Y}_{u,d,e}$	9	$(\mathbf{1}, \mathbf{3}), (\mathbf{2}, \mathbf{0})$	10	$(\mathbf{0}, \mathbf{4})$
Seesaw	$\mathbf{Y}_{u,d,e}, \boldsymbol{\Upsilon}_\nu$	6	$(\mathbf{1}, \mathbf{1})$	7	$(\mathbf{0}, \mathbf{2})$
MSSM	$\mathbf{Y}_{u,d,e}, \boldsymbol{\Upsilon}_\nu$	4	$(\mathbf{1}, \mathbf{1})$	5	$(\mathbf{0}, \mathbf{4})$
		3	$(\mathbf{1}, \mathbf{0}), (\mathbf{0}, \mathbf{1})$	4	$(\mathbf{0}, \mathbf{2})$
SUSY-GUT	$\mathbf{Y}_{10, \bar{5}}$	3	$(\mathbf{1}, \mathbf{0}), (\mathbf{0}, \mathbf{1})$	3	–

Table 1: Leading operators violating baryon ( $\mathcal{B}$ ) or lepton ( $\mathcal{L}$ ) number. The second column indicates the allowed spurion content in each case (see main text for more details). The operators not suppressed by the neutrino masses  $\boldsymbol{\Upsilon}_\nu$  are indicated in bold. The dimensions refer to the Lagrangian terms for the first three scenarios, and to the superpotential terms for the last two. For SUSY-GUT, we take  $SU(5)$  for definiteness, and consider only renormalizable couplings since the usual  $\Delta\mathcal{B}, \Delta\mathcal{L}$  effective interactions suppressed by the GUT scale can always be constructed.

**With four generations,** the Yukawa couplings  $\mathbf{Y}_u, \mathbf{Y}_d$ , and  $\mathbf{Y}_e$  permit to construct  $\Delta\mathcal{B} = \pm 4/N_c$  and  $\Delta\mathcal{L} = \pm 4$  effective operators ( $\Delta\mathcal{L} = \pm 2$  with a Majorana neutrino mass term), but the former are not color singlets when  $N_c = 3$ . Imposing the SM gauge invariance forces the introduction of at least twelve quark fields to violate  $\mathcal{B}$  (see Table 1), since  $n = 3$  is the smallest integer for which  $\Delta\mathcal{B} = \pm 4n/3$  is also an integer. Thus, the minimal spurion content cannot be used to set a natural scale for the effective  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  interactions inducing proton decay. For example, the flavor-symmetry properties of the Wilson coefficients of the dimension-six Weinberg operators (15) are orthogonal to that of the Yukawa couplings. So, if new physics does not introduce drastically new flavor structures, these operators are forbidden, along with any other  $\Delta\mathcal{B} = \pm 1$  operator, and the proton is absolutely stable. Translated in the four-generation MSSM context, the minimal spurion content effectively enforces not only R-parity, but also the near conservation of  $\mathcal{B}$ , so that proton decay and neutron oscillations are forbidden, and the lightest supersymmetric particle is stable. In this sense, the present work provides a motivation for the introduction of a sequential fourth generation, besides the replacement of R-parity by MFV. Together with the current clues from the electroweak and flavor sectors [11, 12], this most simple extension of the SM (or MSSM) appears very appealing.

In summary, the key to proton stability could well be hidden in a highly nongeneric flavor structure for the  $\Delta\mathcal{B}$  or  $\Delta\mathcal{L}$  interactions. With three generations, these couplings are no more (or less) fine-tuned than the known fermion masses and mixings, while with four generations, their drastically different flavor properties rules them out in a natural way. So, the accidental conservation of  $U(1)_{\mathcal{B}}$  and  $U(1)_{\mathcal{L}}$  by the SM Lagrangian seems to be indeed fortuitous, while imposing R-parity on the MSSM appears redundant. Evidently, what is missing in this picture is a dynamical mechanism to enforce the minimality of the MFV prescription at low-energy. The phenomenological successes of this prescription, both accounting for the suppression of proton decay and the absence of new physics effects in  $K$  and  $B$  physics, should act as clear incentives to pursue this goal.

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## A Numerical estimates for the proton decay rate

The operators of Eq. (15) can induce proton decay. When the spurion content is limited to  $\mathbf{Y}_\nu$  and  $\mathbf{Y}_{u,d,e}$ , their Wilson coefficients are predicted, up to  $\mathcal{O}(1)$  factors. The goal of this Appendix is to detail this prediction, and compare it with the current bounds on the proton lifetime. We do not intend to perform a complete analysis and get a numerical estimate for the Wilson coefficient of each possible operator. Rather, our goal is to get the lowest scale  $\Lambda$  compatible with proton decay constraints, assuming it is the same for all the operators in Eq. (15).

Since there are many ways to insert spurions and contract the flavor indices, it may not appear immediately obvious looking at Eq. (15) which operator and flavor structure is most effective at inducing proton decay. To identify this operator, first note that those involving the  $E$  field will not be competitive compared to those with  $L$ , since they necessitate a  $\mathbf{Y}_e$  insertion to connect to the  $SU(3)_L$  space where  $\mathbf{Y}_\nu$  lives. For example, a contraction like

$$\varepsilon^{IJK}(\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{IJ} (E \mathbf{Y}_e)^K, \quad (27)$$

costs at best a tiny factor  $m_\mu/v_d$  since  $E$  cannot be a  $\tau_R^\dagger$  and  $\mathbf{Y}_e = \mathbf{m}_e/v_d$  is diagonal. We thus remain with  $c_1^{IJKL} L^I Q^J Q^K Q^L$  and  $c_2^{IJKL} L^I Q^J D^{\dagger K} U^{\dagger L}$ . Using either the epsilon tensor of  $SU(3)_Q$ ,  $SU(3)_U$ , or  $SU(3)_D$ , the flavor contractions with the least number of Yukawa insertions are:

$$\begin{aligned} c_1^{IJKL} L^I Q^J Q^K Q^L &= a_1 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKL} Q^J Q^K Q^L \\ &\quad + a_2 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKL} (\mathbf{Y}_u Q)^J (\mathbf{Y}_u Q)^K (\mathbf{Y}_u Q)^L \\ &\quad + a_3 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKL} (\mathbf{Y}_d Q)^J (\mathbf{Y}_d Q)^K (\mathbf{Y}_d Q)^L, \end{aligned} \quad (28)$$

$$\begin{aligned} c_2^{IJKL} L^I Q^J D^{\dagger K} U^{\dagger L} &= a_4 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKL} Q^J (D \mathbf{Y}_d)^{\dagger K} (U \mathbf{Y}_u)^{\dagger L} \\ &\quad + a_5 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKL} (\mathbf{Y}_u Q)^J (D \mathbf{Y}_d \mathbf{Y}_u^\dagger)^{\dagger K} U^{\dagger L} \\ &\quad + a_6 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKL} (\mathbf{Y}_d Q)^J D^{\dagger K} (U \mathbf{Y}_u \mathbf{Y}_d^\dagger)^{\dagger L}, \end{aligned} \quad (29)$$

with  $a_i$  some  $\mathcal{O}(1)$  numerical constants. The  $L^I Q^J D^{\dagger K} U^{\dagger L}$  operator is systematically suppressed by light quark mass factors because it involves the quark  $SU(2)_L$  singlets. Indeed, to contribute to proton decay,  $U$  must be an up quark and  $D$  either a down or strange quark. Since  $\mathbf{Y}_u = \mathbf{m}_u V/v_u$  and  $\mathbf{Y}_d = \mathbf{m}_d/v_d$  (see Eq. (5)), the  $a_4$ ,  $a_5$ , and  $a_6$  terms of  $c_2^{IJKL}$  are all suppressed by the tiny  $m_u/v_u$  and/or  $m_{d,s}/v_d$ . Such suppressions clearly remain if more Yukawa spurions are inserted, since all the entries of  $\mathbf{Y}_u^\dagger \mathbf{Y}_u$  and  $\mathbf{Y}_d^\dagger \mathbf{Y}_d$  are smaller than one.

By contrast, the  $LQ^3$  operator involves quark fields of the same type, so they can be immediately contracted into a flavor singlet. One should further keep in mind that the up-quark components of  $Q$  are not mass eigenstates (see Eq. (5)), so  $Q^{2,3}$  do contain the up-quark. The main subtlety is that the  $a_1$  term of  $c_1^{IJKL}$  actually vanishes because  $L^I Q^J Q^K Q^L$  is symmetric under  $L \leftrightarrow K$ , as can be seen writing down the  $SU(3)_C$  and  $SU(2)_L$  antisymmetric contractions explicitly, so we replace it by

$$a'_1 \varepsilon^{IMN} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKO} (\mathbf{Y}_u^\dagger \mathbf{Y}_u)^{OL} Q^J Q^K Q^L. \quad (30)$$

Let us concentrate on this term. As all the entries of  $\mathbf{Y}_u^\dagger \mathbf{Y}_u$  but  $(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^{33} \approx 0.5$  are small, we set  $L, O = 3$  and extract from the  $\bar{\ell}_L^C u_L' \bar{s}_L^C t_L'$  and  $\bar{\nu}_L^C d_L \bar{s}_L^C t_L'$  operators a piece involving only light quarks using  $t_L' = V_{td}^\dagger u_L$ . With  $V_{td} \sim 10^{-2}$ , this suppression is slightly less expensive than using the off-diagonal entry  $(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^{31}$ , proportional to  $V_{ub} \sim 10^{-3}$ . A similar suppression can be derived from

the  $a_2$  contraction, while the  $a_3$  term is necessarily smaller since  $\mathbf{Y}_d$  is diagonal. By the way, this also shows that the dominant proton decay modes are into strange mesons. Those into nonstrange mesons are induced by  $\ell_L c'_L d_L t'_L$ ,  $c'_L = V_{cd}^\dagger u_L$ , and are thus Cabibbo-suppressed.

On the lepton side, the suppression brought in by the neutrino mass spurion is much more important. Setting the reactor and atmospheric mixing angles to  $\theta_{13} = 0^\circ$  and  $\theta_{atm} = 45^\circ$  [1], the background value for the  $\mathbf{Y}_\nu$  spurion can be written as

$$\mathbf{Y}_\nu \approx \frac{1}{v_u} \left( m_\nu \mathbf{1} + \frac{\Delta m_{21} e^{-2i\alpha}}{\sqrt{2}(1+t_\odot^2)} \begin{pmatrix} \sqrt{2}t_\odot^2 & t_\odot & -t_\odot \\ t_\odot & 1/\sqrt{2} & -1/\sqrt{2} \\ -t_\odot & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} + \frac{\Delta m_{31} e^{-2i\beta}}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \right), \quad (31)$$

where  $\theta_\odot$  is the solar mixing angle (we set  $\tan \theta_\odot = 2/3$  in the following),  $\alpha$  and  $\beta$  are the Majorana phases, and the mass-eigenstates are written in terms of an overall scale as  $m_{\nu 1} = m_\nu$ ,  $m_{\nu 2} = m_\nu + \Delta m_{21}$ ,  $m_{\nu 3} = m_\nu + \Delta m_{31}$ . Depending on the spectrum (i.e., whether  $\nu_1$  or  $\nu_3$  is the lightest neutrino), the mass differences  $\Delta m_{21}$  and  $\Delta m_{31}$  are related to the mixing parameters as

$$\Delta m_{21} = (\Delta m_\odot^2 + m_\nu^2)^{1/2} - m_\nu > 0, \quad \begin{cases} \Delta m_{31} = (\Delta m_{atm}^2 + m_\nu^2)^{1/2} - m_\nu > 0 & (\text{Normal}), \\ \Delta m_{31} = (m_\nu^2 - \Delta m_{atm}^2)^{1/2} - m_\nu < 0 & (\text{Inverted}). \end{cases} \quad (32)$$

Therefore, for fixed  $\Delta m_\odot^2 \approx 8 \cdot 10^{-5} \text{ eV}^2$  and  $\Delta m_{atm}^2 \approx 2 \cdot 10^{-3} \text{ eV}^2$  [1], the off-diagonal elements of  $\mathbf{Y}_\nu$  quickly decrease with increasing  $m_\nu$ . Discarding the Majorana phases (irrelevant here), and setting  $\tan \beta = 1$  (to recover the SM Higgs sector), we find for the normal hierarchy

$$\varepsilon^{IJK} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{IJ} \approx \frac{1}{v^3} \begin{pmatrix} \frac{1}{2} m_\tau^2 \Delta m_{31} \\ \frac{1}{2} m_\tau^2 \Delta m_{21} \\ \frac{1}{3} m_\mu^2 \Delta m_{21} \end{pmatrix} \approx \begin{pmatrix} 10^{-17} \rightarrow 10^{-19} \\ 10^{-18} \rightarrow 10^{-21} \\ 10^{-21} \rightarrow 10^{-23} \end{pmatrix} \text{ for } m_\nu = 0 \rightarrow 1 \text{ eV}. \quad (33)$$

Since  $m_\nu$  can vary only between  $(\Delta m_{atm}^2)^{1/2}$  and about 1 eV for the inverted hierarchy, smaller values are found. It is important to note that only neutrino mass differences appear, as the first term of  $\mathbf{Y}_\nu$  gets projected out in the antisymmetric contraction. That is why the suppression is maximal for heavy neutrinos. Another consequence is that  $\varepsilon^{IJK} (\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{IJ}$  has an inverted hierarchy compared to  $\mathbf{Y}_e$ , having its first-generation entry significantly larger. In other words, the dominant proton decay channels are  $p^+ \rightarrow e^+ K^0$  and  $p^+ \rightarrow \bar{\nu}_e K^+$ , as induced by  $\bar{e}_L^C u_L \bar{s}_L^C u_L$  and  $\bar{\nu}_{eL}^C d_L \bar{s}_L^C u_L$  (strictly speaking, the  $\nu_e$  is not a mass eigenstate, but this is of no concern since it is not detected).

The final MFV prediction is thus  $c_1^{eusu} \sim 10^{-19} \rightarrow 10^{-21}$  for  $m_\nu \sim 0 \rightarrow 1 \text{ eV}$  for  $a_1 \sim 1$ . From this, we can estimate the proton decay rate as

$$\Gamma_{p^+} \approx \frac{\alpha_p^2 m_p}{16\pi F_K^2} (1 - m_K^2/m_p^2)^2 \frac{|c_1^{eusu}|^2}{\Lambda^4} \approx 10^{-65} \times a_1^2 \left( \frac{c_1^{eusu}}{10^{-21}} \right)^2 \left( \frac{0.003 \text{ GeV}^3}{\alpha_p} \right)^2 \left( \frac{10 \text{ TeV}}{\Lambda} \right)^4, \quad (34)$$

to be compared with the current limits,  $\Gamma(p^+ \rightarrow e^+ K^0) < 1.4 \cdot 10^{-64}$  and  $\Gamma(p^+ \rightarrow \bar{\nu}_e K^+) < 3.1 \cdot 10^{-65}$ . For the Cabibbo-suppressed mode, the current limit  $\Gamma(p^+ \rightarrow e^+ \pi^0) < 2.5 \cdot 10^{-66}$  is tighter, but leads to a similar scale  $\Lambda$  once accounting for  $c_1^{eudu}/c_1^{eusu} \sim V_{cd}$ . Note that coincidentally, the dominant decay modes are precisely those for which the experimental constraints are the tightest, so if the scale  $\Lambda$  is high enough for them, the bounds for all the other modes are easily satisfied.

The conclusion of this numerical analysis is that the  $\Delta(\mathcal{B} + \mathcal{L})$  scale  $\Lambda$  could be as low as about 10 TeV, to be compared to  $\Lambda \gtrsim 10^{11} \text{ TeV}$  when  $c_1 \sim 1$ . Importantly, this should be understood as

a rough estimate of the true NP scale. First, trivially,  $a_1 \sim 0.1 \rightarrow 10$  would still qualify as natural, and this translates as a factor 1/3 to 3 for  $\Lambda$ . Second, current evaluations for the relevant hadronic matrix element, parametrized by  $\alpha_p$ , range from 0.003 to 0.03  $\text{GeV}^3$ , see Ref. [38], so the high-end of this range would require  $\Lambda$  to be about three times larger. Third, and more importantly, this estimate is purely based on the flavor symmetry and thus misses completely any dynamical effect. The  $LQ^3$  effective operator could be suppressed by some gauge and/or loop factors. For example,  $\Lambda$  is brought down to about 1 TeV if  $a_1 \sim \alpha/4\pi$ . Further, specific NP models need not generate all the effective operators with equal weight. For example, as said in the text,  $LQ^3$  does not arise at tree-level in the R-parity violating MSSM. Since all the other operators are significantly more suppressed by light-quark mass factors, their scale  $\Lambda$  can actually be as low as a few hundred GeV without violating proton decay bounds.

## References

- [1] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37** (2010) 075021.
- [2] P. Nath and P. Fileviez Perez, Phys. Rept. **441** (2007) 191 [arXiv:hep-ph/0601023].
- [3] R. S. Chivukula and H. Georgi, Phys. Lett. B **188** (1987) 99.
- [4] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155; See also e.g. L. J. Hall and L. Randall, Phys. Rev. Lett. **65** (1990) 2939; A. Ali and D. London, Eur. Phys. J. C **9** (1999) 687; A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500** (2001) 161; G. Isidori, F. Mescia, P. Paradisi, C. Smith and S. Trine, JHEP **0608** (2006) 064; R. Zwicky and T. Fischbacher, Phys. Rev. D **80** (2009) 076009; C. Smith, Acta Phys. Polon. Supp. **3** (2010) 53.
- [5] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B **728** (2005) 121; L. Mercolli and C. Smith, Nucl. Phys. B **817** (2009) 1.
- [6] G. Colangelo, E. Nikolidakis and C. Smith, Eur. Phys. J. C **59** (2009) 75;
- [7] E. Nikolidakis and C. Smith, Phys. Rev. D **77** (2008) 015021; C. Smith, Talk given at ICHEP08, Philadelphia, PA, 30 Jul - 5 Aug 2008, arXiv:0809.3152 [hep-ph].
- [8] S. Davidson and S. Descotes-Genon, JHEP **1011** (2010) 073 [arXiv:1009.1998 [hep-ph]].
- [9] C. Csaki, Y. Grossman and B. Heidenreich, arXiv:1111.1239 [hep-ph].
- [10] P. H. Frampton, P. Q. Hung and M. Sher, Phys. Rept. **330** (2000) 263; M. Maltoni, V. A. Novikov, L. B. Okun, A. N. Rozanov and M. I. Vysotsky, Phys. Lett. B **476** (2000) 107; H. J. He, N. Polonsky and S. f. Su, Phys. Rev. D **64** (2001) 053004; B. Holdom, W. S. Hou, T. Hurth, M. L. Mangano, S. Sultansoy and G. Unel, PMC Phys. A **3** (2009) 4.
- [11] See e.g. G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D **76** (2007) 075016; M. S. Chanowitz, Phys. Rev. D **79** (2009) 113008; J. Erler and P. Langacker, Phys. Rev. Lett. **105** (2010) 031801.

- [12] See e.g. M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, Phys. Rev. D **79** (2009) 113006; A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, Phys. Lett. B **683** (2010) 302; Phys. Rev. D **82** (2010) 033009; O. Eberhardt, A. Lenz and J. Rohrwild, Phys. Rev. D **82** (2010) 095006.
- [13] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, C. Promberger and S. Recksiegel, JHEP **1009** (2010) 106.
- [14] See e.g. B. Holdom, JHEP **0608** (2006) 076; J. Alwall *et al.*, Eur. Phys. J. C **49** (2007) 791; G. Eilam, B. Melic and J. Trampetic, Phys. Rev. D **80** (2009) 116003; D. Atwood, S. K. Gupta and A. Soni, arXiv:1104.3874 [hep-ph].
- [15] W. S. Hou, Chin. J. Phys. **47** (2009) 134.
- [16] G. Aad *et al.* [ATLAS Collaboration], arXiv:1109.4725 [hep-ex]; G. Aad *et al.* [ATLAS Collaboration], JHEP **10** (2011) 107; A. Ivanov, [CDF and D0. Collaborations], FPCP 2011 Conference Proceedings, arXiv:1109.1025 [hep-ex]; T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **106** (2011) 141803; T. Aaltonen *et al.* [CDF Collaboration], arXiv:1107.3875 [hep-ex]; V. M. Abazov *et al.* [D0 Collaboration], arXiv:1104.4522 [hep-ex]; T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **106** (2011) 191801; M. M. H. Luk [CMS Collaboration], Proceedings of the DPF-2011 Conference, Providence, RI, August 8-13, 2011, arXiv:1110.3246 [hep-ex]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **701** (2011) 204.
- [17] C. J. Flacco, D. Whiteson, T. M. P. Tait and S. Bar-Shalom, Phys. Rev. Lett. **105** (2010) 111801; H. Murayama, V. Rentala, J. Shu and T. T. Yanagida, Phys. Lett. B **705** (2011) 208.
- [18] G. Aad *et al.* [ATLAS Collaboration], Eur. Phys. J. C **71** (2011) 1728; T. Aaltonen *et al.* [CDF and D0 Collaboration], Phys. Rev. D **82** (2010) 011102; D. Benjamin [for the CDF and D0 and The TEVNPH Working Group Collaborations], Talk given at the EPS 2011 Conference, arXiv:1108.3331 [hep-ex]; D. Trocino [CMS Collaboration], proceedings of the DPF-2011 Conference, Providence, RI, August 8-13, 2011, arXiv:1110.1938 [hep-ex]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **699** (2011) 25.
- [19] J. F. Gunion, arXiv:1105.3965 [hep-ph].
- [20] <http://press.web.cern.ch/press/PressReleases/Releases2011/PR25.11E.html>.
- [21] G. Guo, B. Ren and X. G. He, arXiv:1112.3188 [hep-ph].
- [22] P. Fileviez Perez and M. B. Wise, Phys. Rev. D **82** (2010) 011901 [Erratum-ibid. D **82** (2010) 079901]; T. R. Dulaney, P. Fileviez Perez and M. B. Wise, Phys. Rev. D **83** (2011) 023520.
- [23] Z. Murdock, S. Nandi and Z. Tavartkiladze, Phys. Lett. B **668** (2008) 303.
- [24] P. M. Ferreira, R. Santos, M. Sher and J. P. Silva, arXiv:1112.3277 [Unknown].
- [25] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566; F. Wilczek and A. Zee, Phys. Rev. Lett. **43** (1979) 1571; L. F. Abbott and M. B. Wise, Phys. Rev. D **22** (1980) 2208.
- [26] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 8; Phys. Rev. D **14** (1976) 3432 [Erratum-ibid. D **18** (1978) 2199].

- [27] P. Minkowski, Phys. Lett. B **67** (1977) 421; M. Gell-Mann, P. Ramond, R. Slansky, Supergravity Stony Brook Workshop, New York, 1979; T. Yanagida, Workshop on the Baryon Number of the Universe & Unified Theories, Tsukuba, Japan, 1979; S. L. Glashow, Quarks & Leptons, Cargèse, 1979; R. N. Mohapatra, G. Senjanovic, Phys. Rev. D **23** (1981) 165.
- [28] G. Burdman, L. Da Rold and R. D. Matheus, Phys. Rev. D **82** (2010) 055015; A. Lenz, H. Pas and D. Schalla, arXiv:1104.2465 [hep-ph].
- [29] R. Barbier *et al.*, Phys. Rept. **420** (2005) 1.
- [30] G. R. Farrar and P. Fayet, Phys. Lett. B **76** (1978) 575.
- [31] L. E. Ibanez and G. G. Ross, Nucl. Phys. B **368** (1992) 3.
- [32] B. Grinstein, V. Cirigliano, G. Isidori and M. B. Wise, Nucl. Phys. B **763** (2007) 35.
- [33] P. Q. Hung, Phys. Rev. Lett. **80** (1998) 3000.
- [34] R. M. Godbole, S. K. Vempati and A. Wingerter, JHEP **1003** (2010) 023.
- [35] B. Holdom, Phys. Rev. Lett. **57** (1986) 2496 [Erratum-ibid. **58** (1987) 177]; S. F. King, Phys. Lett. B **234** (1990) 108; C. T. Hill, M. A. Luty and E. A. Paschos, Phys. Rev. D **43** (1991) 3011; R. Fok and G. D. Kribs, Phys. Rev. D **78** (2008) 075023; P. Q. Hung and C. Xiong, Nucl. Phys. B **847** (2011) 160; Phys. Lett. B **694** (2011) 430.
- [36] A. Y. Smirnov and F. Vissani, Nucl. Phys. B **460** (1996) 37.
- [37] For a review, see e.g. R. N. Mohapatra, Lectures at TASI 1997: Supersymmetry, Supergravity and Supercolliders, Boulder, CO, 1-7 Jun 1997, arXiv:hep-ph/9801235.
- [38] Y. Aoki, C. Dawson, J. Noaki and A. Soni, Phys. Rev. D **75** (2007) 014507.